National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-3760

ST - CM - GC - 10346

NASA TT F-9688

RENDEZVOUS PROBLEM OF SPACE VEHICLE WITH ORBITAL STATION

by

G. Yu. Markov
F. A. Mikhaylov

[USSR]

21 JUNE 1965

NASA TT F-9688
ST — CM — GC — 10346

RENDEZVOUS PROBLEM OF SPACE VEHICLE WITH ORBITAL STATION

Preprint of a Paper
to be presented at the
International Federation of
Automatic Control (IFAC)
Stavanger, Norway, 21 — 25 June 1965

by G. YU. Markov &
F. A. Mikhaylov

33816

SUMMARY

Considering that the various schemes of spacecraft rendezvous with orbital station consist of three stages, and that either of these stages consists in its turn of three phases, the present work is devoted to the problem of spacecraft guidance at long-range rendezvous phase with orbital station, utilizing the second derivative of the relative remoteness for information on line-of-sight-rotation [1]. author

INTRODUCTION AND PROBLEM DEFINITION

The problem of convergence and rendezvous in orbit has attracted in the course of the past few years the attention of numerous specialists. As a result, there emerged a substantial number of organizational schemes for spacecraft rendezvous process with orbital station. These schemes can be subdivided into three stages [1], [2], [3] :

1.- Direct launching of spacecraft into the orbit plane of the space station.

2.- Utilization of phasing ellipse or of parking orbit.

3.- Noncoplanar flight.

---

* NEKOTORYYE VOPROSY SBLIZHENIYA S ORBITAL'NOY STANTSIEY.

In its turn, any of the indicated rendezvous schemes consist of three phases:

1.- The active region.

2.- The passive flight region.

3.- The escape region to station's orbit and maneuvers required for rendezvous materialization.

The guidance is also realized correspondingly with the three indicated phases for rendezvous.

Thus, in the first phase, the problem resolved is, in essence, that of bringing out the spacecraft into the pre-assigned orbit.

In the second phase, guidance is reduced to the selection of its course from the condition of collision with the station without applying thrust [1]. - This phase may be called long-range guidance.

The third phase consists in the materialization of convergence velocity decrease to zero and of coasting. At times, this phase is referred to as short-range guidance.

The present work is concerned with the problem of spacecraft guidance in the long-range rendezvous phase with the orbital station, utilizing the second derivative of relative remoteness for information on line-of-sight rotation.

The condition that has to be satisfied by the relative motion of spacecraft and orbital station in the rendezvous phase under consideration, consists in that the relative velocity vector coincides as precisely as possible with the line of sight [1].

1. EQUATIONS OF SPACECRAFT'S RELATIVE MOTION

Let us consider a rectangular inertial system of coordinates (X, Y, Z) with origin at the Earth's mass center, whose axes form the right-handed system (Fig. 1).

Two point move in that system: the point 1, that is, the spacecraft (X_1, Y_1, Z_1) ; point 2 — the orbital station (X_2, Y_2, Z_2).

We shall construct a system of coordinates (ξ, η, ζ) with the center at the point (X_1, Y_1, Z_1), so that the axis of both systems are parallel [4].

Let us consider the motion of the point (X_2, Y_2, Z_2) in the system (ξ, η, ζ), that the motion of the former relative to the point (X_1, Y_1, Z_1). The coordinates of the point (X_2, Y_2, Z_2) are linked in the relative system of coordinates with the coordinates in the inertial system by the following correlations

$$\begin{aligned} \xi &= X_2 - X_1, \\ \eta &= Y_2 - Y_1, \\ \zeta &= Z_2 - Z_1. \end{aligned} \qquad (1)$$

Upon differentiating these correlations twice, we may write the equations of motion in the relative system of coordinates $(\xi \eta \zeta)$ as follows:

$$\begin{aligned} \ddot{\xi} &= W_{2X} - W_{1X}, \\ \ddot{\eta} &= W_{2Y} - W_{1Y}, \\ \ddot{\zeta} &= W_{2Z} - W_{1Z}, \end{aligned} \qquad (2)$$

where

$$\begin{aligned} W_{iX} &= \frac{1}{m_i} \sum F_{jX}, \\ W_{iY} &= \frac{1}{m_i} \sum F_{jY}, \\ W_{iZ} &= \frac{1}{m_i} \sum F_{jZ} \end{aligned}$$
$$(i = 1, 2)$$

are the equations of motion of the points in the inertial system of coordinates under the action of the pre-assigned systems of forces.

The investigations of the relative motion could be conveniently conducted in the spherical system of coordinates. Let us write the equations in the spherical system of coordinates $(D \varepsilon_1 \varepsilon_2)$ (see Fig. 2).

The equations of link have the form :

$$\begin{aligned} \xi &= D \cos \varepsilon_1 \cos \varepsilon_2, \\ \eta &= D \sin \varepsilon_1, \\ \zeta &= D \cos \varepsilon_1 \sin \varepsilon_2. \end{aligned} \qquad (3)$$

The equations of relative motion in the spherical system of coordinates have the form :

../..

$$\ddot{D} - D\dot{\varepsilon}_1^2 - D\cos^2\varepsilon_1\,\dot{\varepsilon}_2^2 = (W_{2X} - W_{1X})\cos\varepsilon_1\cos\varepsilon_2 + (W_{2Y} - W_{1Y})\sin\varepsilon_1 + (W_{2Z} - W_{1Z})\cos\varepsilon_1\sin\varepsilon_2\,,$$

$$\frac{1}{D}\cdot\frac{d}{dt}\; D^2\dot{\varepsilon}_1 + D\sin\varepsilon_1\cos\varepsilon_1\,\dot{\varepsilon}_2^2 = (W_{2X} - W_{1X})\sin\,\varepsilon_1\cos\,\varepsilon_2 + (W_{2Y} - W_{1Y})\cos\,\varepsilon_1 - (W_{2Z} - W_{1Z})\sin\varepsilon_1\sin\varepsilon_2, \tag{4}$$

$$\frac{1}{D\cos\varepsilon_1}\cdot\frac{d}{dt}\left(D^2\cos^2\,\varepsilon_1\,\dot{\varepsilon}_2\right) = -(W_{2X} - W_{1X})\sin\,\varepsilon_2 + (W_{2Z} - W_{1Z})\cos\,\varepsilon_2$$

Let us consider at further length the following particular case of relative motion.

Assume that the spacecraft and the orbital station are situated in free flight. Assume also, that the relative remoteness is so small that the influence of the gravitational field upon the relative motion can be neglected. Then, the equations (2) will take the form:

$$\ddot{\xi} = \ddot{\eta} = \ddot{\zeta} = 0 \tag{5}$$

and the equations (4) will be written as follows :

$$\begin{aligned} &\ddot{D} - D\,\dot{\varepsilon}_1^2 - D\cos^2\varepsilon_1\,\dot{\varepsilon}_2^2 = 0 \\ &\frac{1}{D}\cdot\frac{d}{dt}\left(D^2\dot{\varepsilon}_1\right) + D\sin\varepsilon_1\cos\varepsilon_1\,\dot{\varepsilon}_1^2 = 0 \\ &\frac{1}{D\cos\varepsilon_1}\cdot\frac{d}{dt}\left(D^2\cos^2\,\varepsilon_1\,\dot{\varepsilon}_2\right) = 0 \end{aligned} \tag{6}$$

Integrating the equations (6), we shall obtain the expression for the relative remoteness

$$D^2 = \bar{V}_o^2\left(t + \frac{D_o\dot{D}_o}{\bar{V}_o^2}\right)^2 + \frac{V_{n1o}^2 + V_{n2o}^2}{\bar{V}_o^2}\,D_o^2\,, \tag{7}$$

where

$$\bar{V}_o^2 = \dot{D}_o^2 + V_{n1o}^2 + V_{n2o},$$

here $\bar{V}_o$ is the relative velocity at a certain zero moment of time, D_o and $\dot{D}_o$ are respectively the relative remoteness and the velocity of convergence at the same moment of time, V_{n1} and V_{n2} are the relative velocity components in a plane perpendicular to the relative remoteness (line of sight), which are linked with the spherical coordinates as follows :

$$V_{n1} = D\dot{\varepsilon}_1 ,$$
$$V_{n2} = D\cos\varepsilon_1\dot{\varepsilon}_2 .$$

V_{n10} and V_{n20} are their values at the moment of time zero.

One of the first integrals of the system (6) is the energy integral

$$\bar{V}_o^2 = \dot{D}^2 + V_{n1}^2 + V_{n2}^2 \qquad (8)$$

It follows from formula (7) that if $V_{n10} \neq 0$ and $V_{n20} \neq 0$, the converging devices will attain the least relative distance at the moment of time

$$T = \frac{D_o\dot{D}_o}{\bar{V}_o^2}$$

and then will begin to drift away.

The square of that distance is

$$h_{np}^2 = \frac{V_{n10}^2 + V_n{}^2{}_0}{\bar{V}_o^2}\dot{D}_o^2 \qquad (9)$$

2. - ON THE UTILIZATION OF THE SECOND DERIVATIVE OF REMOTENESS FOR THE INFORMATION ON SIGHTING LINE ROTATION

We examined in the first chapter the model of relative motion in a force-free field. If the spatial device (spacecraft) is in free flight, or moves under the action of an active force, applied along the normal to the line of sight, the first equation of the system (6) is valid:

$$\ddot{D} - D\dot{\varepsilon}_1^2 - D\cos^2\varepsilon_1\dot{\varepsilon}_2 = 0$$

Multiplying this equation by D, we shall obtain

$$D\ddot{D} = D^2\dot{\varepsilon}_1^2 - D^2\cos^2\varepsilon_1\dot{\varepsilon}_2^2 \qquad (10)$$

The right-hand part of the equation (10) represents the square of the velocity component perpendicular to the line of sight.

We have in truth:

$$V_{n1} = D\dot{\varepsilon}_1$$
$$V_{n2} = D\cos\varepsilon_1\dot{\varepsilon}_2$$
$$D\ddot{D} = V_{n1}^2 + V_{n2}^2 \qquad (11)$$

It follows from the equation (11), that at $\ddot{D} = 0$, $V_{n1} = V_{n2} = 0$, that is, the conditions for parallel convergence are satisfied. Inversely, at $V_{n1} = V_{n2} = 0$, we shall have $\ddot{D} = 0$.

Therefore, the second derivative of relative remoteness can serve as the source of information on line-of-sight rotation in a force-free field, istead of the angular velocity.

Note one interesting property of the parameter $\ddot{D}$. As follows from the equation (11), parameter $\ddot{D}$ cannot take negative values, that is, we always have $\ddot{D} \geqslant 0$.

Let us rewrite the equation (11) in the form

$$\ddot{D} = \frac{\tilde{V}^2}{D} - \frac{\dot{D}^2}{D}$$

where $\tilde{V}$ - is the relative velocity.

It follows from this last equation that for a given value of the relative velocity, the greatest value of $\ddot{D}$ is obtained at time of fulfillment of the condition $\dot{D} = 0$, that is, at time of attaining the minimum distance between the converging bodies, and at very great relative remoteness $\ddot{D}$ is near zero, that is, at $D \to \infty$, the second derivative of the relative remoteness approaches zero.

Fig. 3 shows the variation of parameters $\dot{\varepsilon}_1$, $\ddot{D}$, and $D\ddot{D}$ with time.

From the first equation of the system (6) we may obtain the correlation

$$\delta D = 2V_{n1}\,\delta\dot{\varepsilon}_1 + 2V_{n2}\cos\varepsilon_1\delta\dot{\varepsilon}_2 \qquad (12)$$

which allows to compare among themselves the measurement precisions of angular velocity signals and of the second derivative of remoteness.

The partial derivatives

$$\frac{\partial \ddot{D}}{\partial \dot{\varepsilon}_1} = 2\,V_{n1} \quad \text{and} \quad \frac{\partial \ddot{D}}{\partial \dot{\varepsilon}_2} = 2\,V_{n2}\cos\varepsilon_1$$

are proportional to the relative velocity component in a plane perpendicular to the line of sight.

It was pointed out above, that at the long-range rendezvous phase the guided spacecraft homing must be effected in such a way, that the relative velocity vector coincide as precisely as possible with the line of sight.- Usually, for the solution of this problem the guiding action is directed along the normal to the line of sight, so that the problem of guidance amounts to the stabilization of the line of sight in the inertial space (X, Y, Z), the guiding signal being the angular velocity of the line of sight. If we utilize the second derivative of relative remoteness for information on line-of-sight rotation, while the guiding action, induced by the engine of spacecraft, is directed along the line of sight in this case too, the equations of relative motion will be :

$$\ddot{D} - D\dot{\varepsilon}_1^{\,2} - D\cos^2\varepsilon_1\,\dot{\varepsilon}_2^{\,2} = 0$$

$$\frac{1}{D}\,\frac{d}{dt}\left(D^2\dot{\varepsilon}_1\right) + D\sin\varepsilon_1\cos\varepsilon_1\,\dot{\varepsilon}_2^{\,2} = f_1(\ddot{D}), \qquad (13)$$

$$\frac{1}{D\cos\varepsilon_1}\,\frac{d}{dt}\left(D^2\cos^2\varepsilon_1\,\dot{\varepsilon}_2^{\,2}\right) = f_2(\ddot{D}),$$

where $f_1(\ddot{D})$ and $f_2(\ddot{D})$ are the guiding accelerations acting upon the spacecraft.

Therefore, it is possible to construct a closed guidance system at the long-range rendezvous phase by merely conducting measurements of the second derivative of relative remoteness with, for example, the aid of the Doppler effect, and utilizing the information obtained for the formulation of the appropriate guidance. It should be noted at the same time, that one channel (the line of sight) is utilized to obtain the information about the relative motion, while the other two, perpendicular to the first, serve to achieve guidance by the motion of spacecraft's center of masses.

**** THE END ****

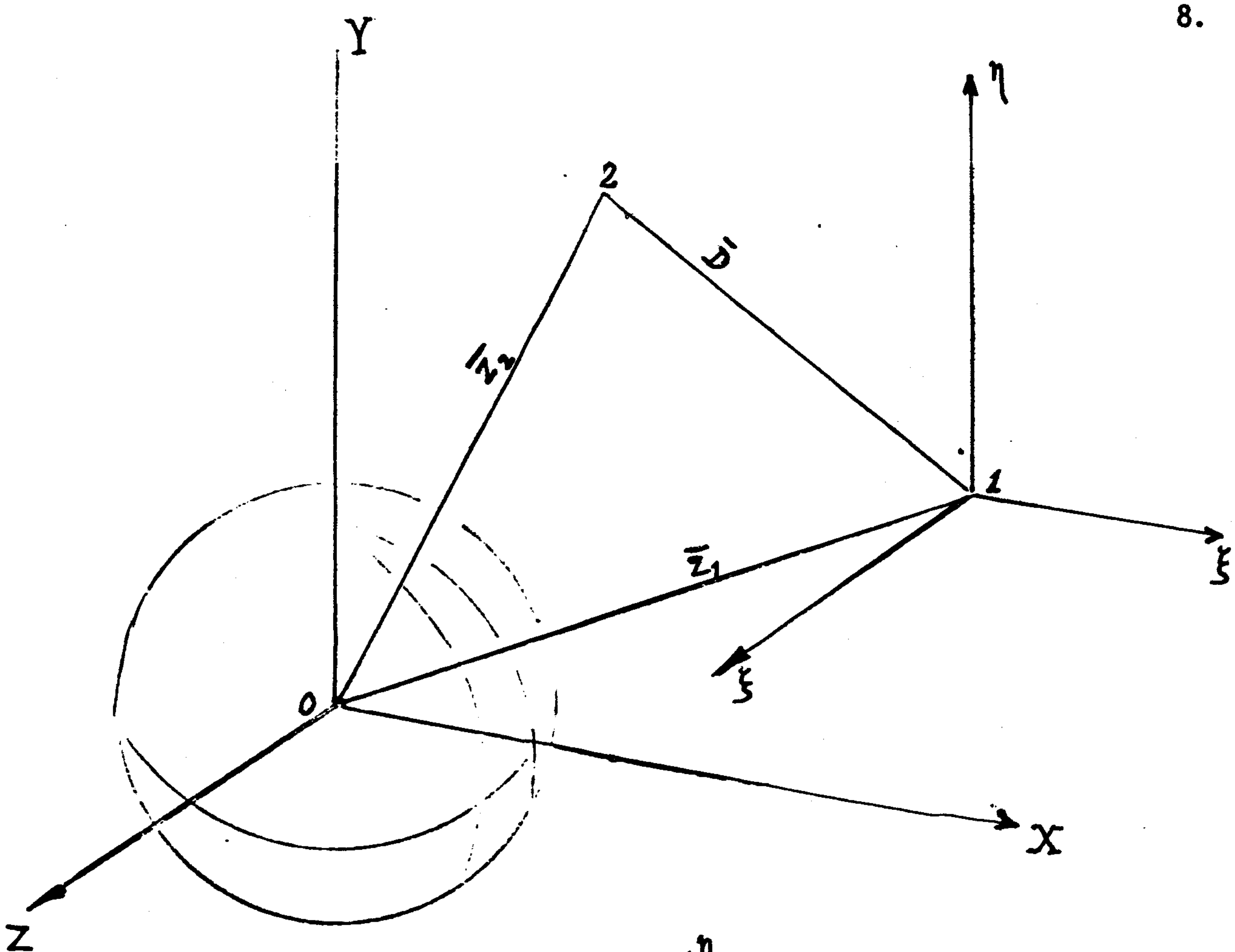


Fig. 1

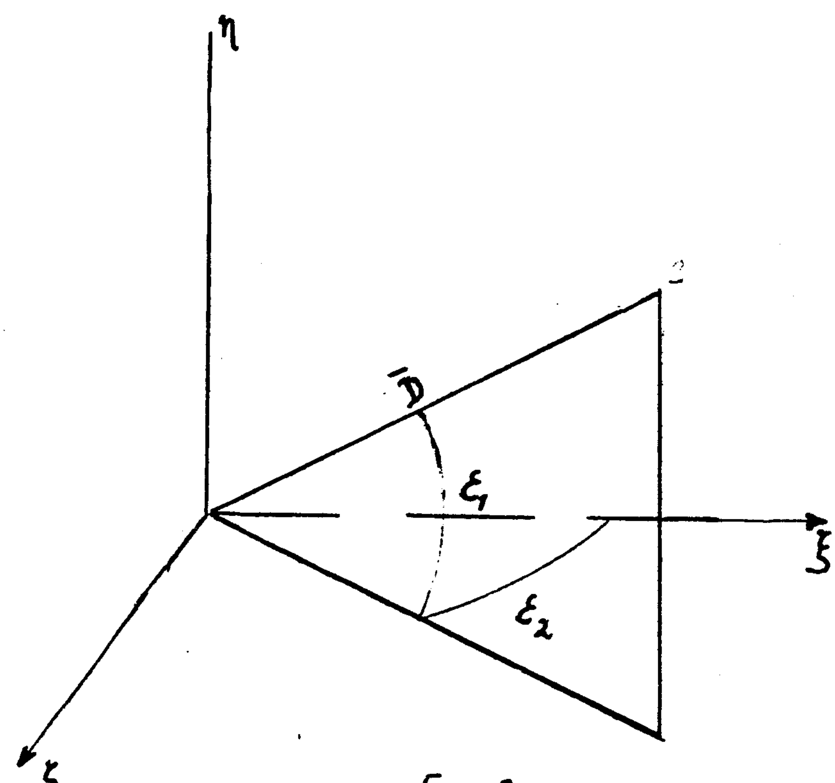
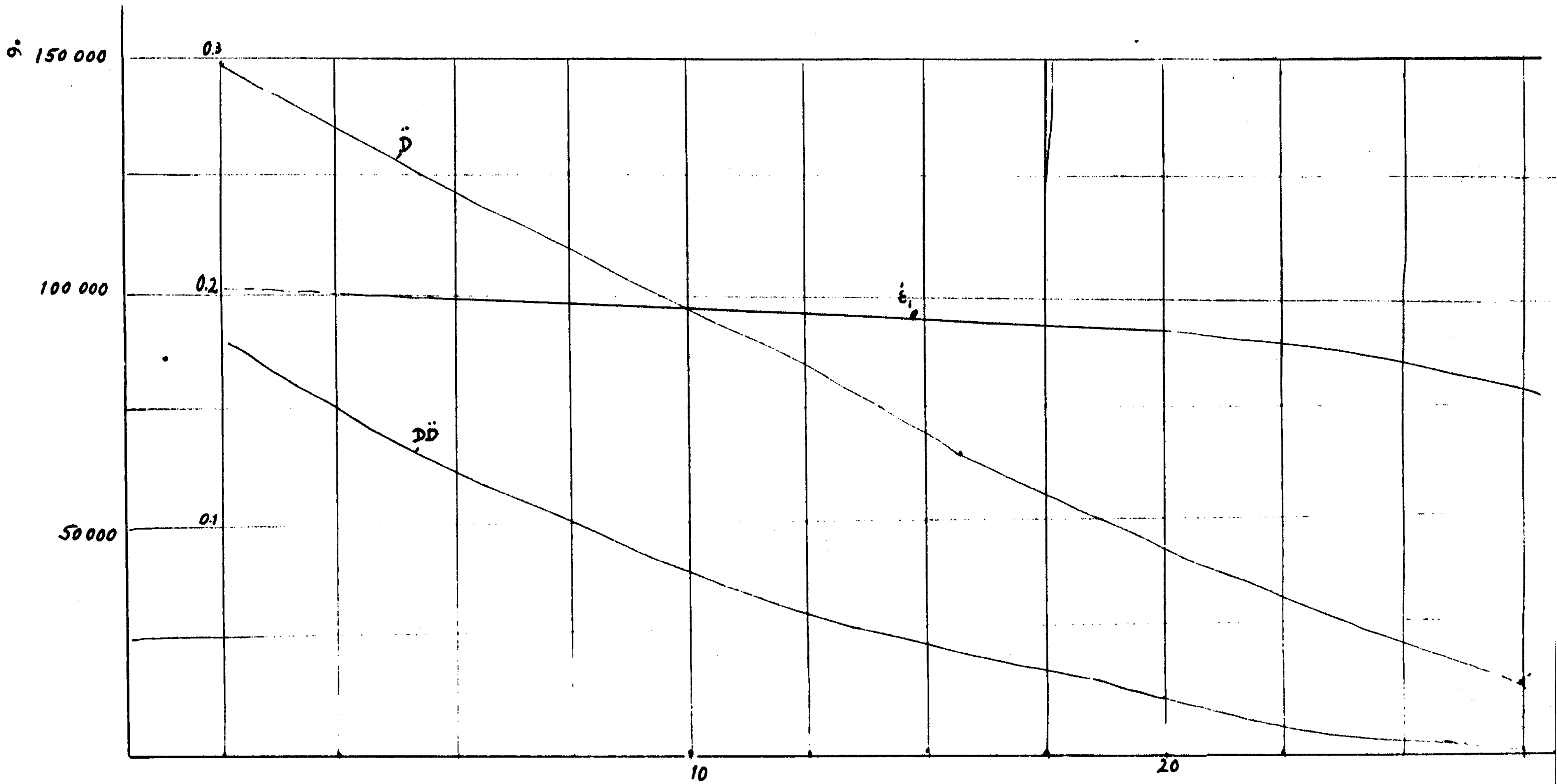


Fig. 2



150 000
100 000
50 000
0.3
0.2
0.1
$\ddot{D}$
$\dot{\varepsilon}_i$
$D\ddot{D}$
10
20

Fig. 3

REFERENCES

[1].- PETER W. SOULE, ALAN T. KIDD. Terminal Maneuvers for Satellite Ascent Rendezvous.- Prepr. ARS, 206, 28, 1961.

[2].- W. H. STRALY.- The Phasing Techniques in Rendezvous.- ARS J. 32, No. 4, pp 620-6, 1962.

[3].- V. PONOMAREV.- Vstrecha na orbite (Rendezvous in Orbit).- Aviatsiya i Kosmonavtika, No. 2, 27-32, 1963.

[4].- J. HOUBOLT.- Problems and Potentialities of Space Rendezvous. J. Astronautica Acta Vol.VII, Fasc. 5-6, 1961.

Contract No. NAS-5-3760
Consultants and Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT
on 17 — 18 June 1965

DISTRIBUTION

GODDARD SPACE F.C.

540	FLEMING	
541	RICHARDSON	
542	GORMAN	
547	SIRY	
	WAGNER	
600	TOWNSEND	
	STROUD	
610	MEREDITH	
611	McDONALD	
612	HEPPNER	
613	KUPPERIAN	
614	LINDSAY, WHITE	
615	BOURDEAU	
640	HESS	[3]
643	SQUIRES	
	LIU	
	SHUTE	
660	GI for SS	[5]
252	LIBRARY	[2]
256	FREAS	

N A S A H Q S

SS	NEWELL	
	GARBARINI	
SG	NAUGLE	
	ROMAN	
	SCHARDT	
	DUBIN	
SL	NICKS	
	MIWITZKI	
	BRUNK	
SM	FOSTER	
	GILL	
	BADGLEY	
ST	JAFFE	
M	MUELLER	[3]
RR	KURZWEG	
REC	GOULD	
REG	GILCHRIST	
RTR	NEILL	
ATSS	SCHWIND	[4]
	ROBBINS	
WX	SWEET	

OTHER CENTERS

AMES R.C.

SONETT
KURKOWSKI
K. G. GROVER
LESSING
R. E. COATE
LIBR. [3]

LANGLEY

213	KATZOFF	
	BUGLIA	
304	BIRD	
	QUEIJO	
185	WEATHERWAX	[3]

M S C HOUSTON

OSO HUSS

MARSHALL

SCMIEDER
WINCH

UCLA

KAULA

LEWIS